

Dilution of zero point energies in the cosmological expansion

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Abstract

The vacuum fluctuations of all quantum fields filling the universe are supposed to leave enormous energy and pressure contributions which are incompatible with observations. It has been recently suggested that, when the effective nature of quantum field theories is properly taken into account, vacuum fluctuations behave as a relativistic gas rather than as a cosmological constant. Accordingly, zero-point energies are tremendously diluted by the universe expansion but provide an extra contribution to radiation energy. Ongoing and future cosmological observations could offer the opportunity to scrutinize this scenario. The presence of such additional contribution to radiation energy can be tested by using primordial nucleosynthesis bounds or measured on Cosmic Background Radiation anisotropy.

Any phenomenological quantum field theory is an effective field theory (EFT) having a limited energy range of validity. Quantum electrodynamics as well as the 4-fermion theory of weak interactions, for instance, are low energy manifestations of a higher energy theory, namely the Standard Model (SM), which in turn is the low energy limit of an even higher energy model. Nowadays, a large amount of experimental and theoretical work is devoted to the search of the ultraviolet (UV) completion of the SM.

An EFT is intrinsically defined with a physical UV cut-off Λ , the scale which signals the onset of “new physics”. Above Λ the theory is no longer valid and has to be replaced by a higher energy one. This results in a hierarchy of theories each having higher and higher UV cut-off [1]. This hierarchical structure of elementary particle models is believed to survive up to the Planck scale M_P . In the present work, in order to take a more general point of a view, we will keep working with a generic cut-off Λ .

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This naturally leads to the idea that, whatever theory describes physics before a specific time t_Λ , for $t > t_\Lambda$ physics is appropriately described by an EFT with physical cutoff Λ . It has been recently shown that a generic consequence of the effective nature of this EFT (irrespective of the specific model) is that the equation of state (EOS) for the vacuum energy has the unexpected form $p_{vac} = \rho_{vac}/3$, as for a gas of relativistic particles [2].

This can be illustrated by considering a single component real scalar field theory

$$S[\phi] = \int d^4x \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 \right) \quad (1)$$

and computing the quantum-statistical average (the thermal average for a statistical equilibrium distribution) of the corresponding energy-momentum operator,

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \mathcal{L} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left(\frac{1}{2} \partial_\sigma \phi \partial^\sigma \phi - \frac{1}{2} m^2 \phi^2 \right). \quad (2)$$

In fact, for the vacuum contributions to the averages $\langle T_{00} \rangle$ and $\langle T_{ii} \rangle$ (the non-diagonal terms vanish) we find (see [2] for details):

$$\langle T_{00}^{vac} \rangle = \frac{1}{V} \sum_{\vec{k}} \frac{\sqrt{\vec{k}^2 + m^2}}{2} \quad (3)$$

$$\langle T_{ii}^{vac} \rangle = \frac{1}{V} \sum_{\vec{k}} \frac{(k^i)^2}{2 \sqrt{\vec{k}^2 + m^2}}, \quad (4)$$

where V is the quantization volume. By performing in Eqs.(3) and (4) the sum over \vec{k} up to the UV cutoff Λ , the vacuum contributions to the energy density $\rho_{vac} = \langle T_{00}^{vac} \rangle$ and pressure $p_{vac} = \langle T_{ii}^{vac} \rangle$ (due to rotational invariance $\langle T_{11} \rangle = \langle T_{22} \rangle = \langle T_{33} \rangle$) turn out to be:

$$\rho_{vac} = \frac{1}{16\pi^2} \left[\Lambda(\Lambda^2 + m^2)^{\frac{3}{2}} - \frac{\Lambda m^2 (\Lambda^2 + m^2)^{\frac{1}{2}}}{2} - \frac{m^4}{4} \ln \left(\frac{(\Lambda + (\Lambda^2 + m^2)^{\frac{1}{2}})^2}{m^2} \right) \right], \quad (5)$$

$$p_{vac} = \frac{1}{16\pi^2} \left[\frac{\Lambda^3 (\Lambda^2 + m^2)^{\frac{1}{2}}}{3} - \frac{\Lambda m^2 (\Lambda^2 + m^2)^{\frac{1}{2}}}{2} + \frac{m^4}{4} \ln \left(\frac{(\Lambda + (\Lambda^2 + m^2)^{\frac{1}{2}})^2}{m^2} \right) \right]. \quad (6)$$

By noting that $\Lambda \gg m$, we immediately see that in Eqs. (5) and (6) the dominant terms are the first ones, so we have:

$$p_{vac} \sim \frac{\rho_{vac}}{3}. \quad (7)$$

It could be objected that the divergences have no physical meaning and that the definition of a theory has to be completed by some appropriate renormalization procedure which allows to remove them, so that Λ should be regarded just as a regulator.

As mentioned above, however, from a deeper physical point of view, it is more satisfactory to consider a quantum field theory as an effective theory *intrinsically* defined with the help of an UV cut-off Λ and valid up to this scale (see for instance [3]). From this perspective, the cut-off Λ is physical and the consequences of its presence in the very definition of the theory have to be seriously taken into account.

In this respect, it should be recalled that the physical meaning of the quartic divergence Λ^4 which originates from zero point energies is deeply rooted in the underlying harmonic oscillator structure of a quantum field theory. As pointed out in [4], if we cancel out those terms with the help of a formal procedure such as normal ordering, this property is automatically lost.

In passing, it is interesting to notice that in recent years it has been claimed that dark energy, more specifically zero point energies of fundamental fields, can be (and has been) observed in experiments which measure the spectral function of the noise current in resistively shunted Josephson junctions [5, 6]. A thorough analysis of the problem, however, has shown that these claims were based on a misunderstanding of the physical origin of the spectral function [7, 8, 9, 10, 11, 12]. In the present work, on the contrary, the footprint of zero point energies will be searched for in a different class of experiments (WMAP, Planck).

Going back to our original problem, we note that the importance of the quantum field theoretic contribution to the energy-momentum tensor that appears in the Einstein equations,

$$G_{\mu\nu} - \Lambda_{CC} g_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad (8)$$

was firstly recognized in [13] and [14], where however, in accordance with the idea that the divergences are unphysical and have to be discarded, the divergent terms were removed with the help of a renormalization procedure (Pauli-Villars regulators were used in [14]). Such a formal approach is thoroughly analyzed and criticized by De Witt [4]. Still, a popular prescription (often used nowadays) for the automatic (yet formal) cancellation of these divergences is the dimensional regularization scheme (see, for instance, [15]).

In view of the above considerations, in the present paper we follow the approach of [2] and adopt the following “EFT point of view”: we assume that at a given time t_Λ physics is described by an EFT with cut-off Λ .

Scope of this work is to explore some of the phenomenological consequences of this assumption. In particular, we show that the resulting scenario does not conflict with well known results of standard cosmology and that in the very near future it will be possible to test it against experimental data.

Actually, when the renormalized (as opposed to effective) theory is considered, i.e. when the divergent terms are discarded with the help of a specific renormalization scheme (dimensional regularization, Pauli-Villars regulators, ...), the coefficient w in the EOS $p_{vac} = w \rho_{vac}$ turns out to be $w = -1$ [14, 15] (see also the above Eqs. (5) and (6) with the quartic and quadratic divergent terms subtracted). We then conclude that ρ_{vac} does not evolve with time and can be interpreted as a contribution to the cosmological constant. This is the standard view.

Within our effective field theory approach, on the contrary, we keep the large but finite terms proportional to Λ^4 and Λ^2 and therefore we obtain a different EOS, Eq.(7), with $w \sim 1/3$, which is the same as for the relativistic matter case. The immediate implication of this is that the zero point energy density of a quantum field red-shifts with time.

This result suggests the possibility of considering the following scenario, the theoretical aspects of which have been studied in [2].

First of all, we assume that at the time $t \sim t_\Lambda$ and energy scale $E \sim \Lambda$ physics is entirely described by a quantum field, a scalar field for example, and that the lower energy theories were born during the cosmic time evolution. This assumption appears natural in view of our ideas on the effective nature of particle physics theories and fits our current views on the cosmological evolution. Accordingly, the zero point energy density associated to the quantization of this field at $t = t_\Lambda$ is

$$\rho_{vac}(t_\Lambda) = \frac{\Lambda^4}{16\pi^2} \quad (9)$$

where again we have taken $\Lambda \gg m$.

Second we note that, from this perspective, the lower energy fields, i.e. lower energy degrees of freedom (dof), are nothing but a convenient way to parametrise the theory at a lower scale. Therefore, when computing the vacuum contribution to the energy density, one should not include the zero point energies of the effective low energy theories as this would result in a multiple counting of dof. The only contribution from zero point energies to the energy density of the universe comes from the dof of the original theory.

According to this scenario, due to Eq. (7), the zero-point energy contribution to the energy density of the universe, which at t_Λ is $\rho_{vac}(t_\Lambda) \sim \Lambda^4$ (see Eq. (9)), is tremendously diluted by the cosmic evolution. In other words, according to our picture, the zero point energies of an effective field theory (with any mass term neglected) are expressed by the parameter Λ , which corresponds to the upper energy scale cut-off of the effective theory itself. Then, due to their EOS, Eq. (7), which is equal to the EOS for radiation, the zero point energies undergo the same renowned rescaling of radiation energy density with cosmic time.

Clearly, this model does not explain the origin of the measured dark energy (which has the experimentally estimated value $w \sim -1$). However it has the virtue of proposing a natural solution for the theoretical conundrum of the 120 order of magnitude excess of energy density coming from the vacuum fluctuations of quantum fields.

At the same time, this red-shifted zero-point energy provides an additional contribution to the radiation energy density (photons, neutrinos), as can be expected from the EOS, Eq. (7).

Ongoing (as well as future) cosmological observations (will) offer the opportunity to test this scenario against experiments. As we shall see, an important result of our present analysis is that such an excess of radiation-like energy density does not screw up well tested theoretical predictions of standard cosmology and, at the same time, can be compared with present and forthcoming experimental results.

In order to allow for such a comparison, let us consider the usual parametrization for

the total amount of radiation energy density that we are considering at a generic time t after neutrino decoupling [16, 17],

$$\rho_{rad}(t) = \rho_\gamma(t) + \rho_\nu(t) + \rho_x(t) = \left(1 + \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} N_{eff}\right) \rho_\gamma(t) , \quad (10)$$

with N_{eff} given by $N_{eff} = 3 + \delta n_x$, where 3 is the standard number of neutrinos and δn_x accounts for possible extra dof, so called neutrino equivalent dof. Hence, in order to calculate the contribution of zero-point energy contribution to δn_x , we have to evolve ρ_{vac} from t_Λ down to t .

Let us remind that, since ρ_{vac} is a radiation-like term, one can use the following equation to extrapolate the values to the present time t_0

$$\frac{\rho_{vac}(t)}{\rho_\gamma(t)} = \frac{\rho_{vac}(t_0)}{\rho_\gamma(t_0)} \quad (11)$$

Furthermore, as shown in [2], ρ_{vac} at present time is given by

$$\rho_{vac}(t_0) = \rho_{vac}(t_\Lambda) \left(\frac{t_\Lambda}{t_0}\right)^2 \frac{1}{1 + z_{eq}} \quad (12)$$

By using Eqs. (9), (11) and (12), we can write

$$\frac{\rho_{vac}(t)}{\rho_\gamma(t)} = \frac{\rho_{vac}(t_\Lambda)}{\rho_\gamma(t_0)} \left(\frac{t_\Lambda}{t_0}\right)^2 \frac{1}{1 + z_{eq}} = \frac{135}{64 \pi^4} \left(\frac{M_P H_0}{T_0^2}\right)^2 \frac{1}{1 + z_{eq}} \left(\frac{\Lambda^2 t_\Lambda}{M_P}\right)^2 , \quad (13)$$

where z_{eq} is the value of the redshift at the time of matter-radiation equality, while H_0 and T_0 are the Hubble constant and the temperature of the relic photons at present time t_0 respectively. Note that all the terms on the r.h.s. of the previous expression are known except the product $\Lambda^2 t_\Lambda$, which is the free parameter of our model. By replacing Eq. (13) in Eq. (10) and by taking $M_P = 1.22 \cdot 10^{19} \text{ GeV}$, $T_0 = 2.725 \text{ K}$, $H_0^{-1} = 1.27 \cdot 10^{26} \text{ m}$, [18], and $z_{eq} = 3253$, [19], we get :

$$\delta n_x = \frac{8}{7} \left(\frac{11}{4}\right)^{4/3} \frac{135}{64 \pi^4} \left(\frac{M_P H_0}{T_0^2}\right)^2 \frac{1}{1 + z_{eq}} \left(\frac{\Lambda^2 t_\Lambda}{M_P}\right)^2 \sim 3.46 \left(\frac{\Lambda^2 t_\Lambda}{M_P}\right)^2 . \quad (14)$$

We turn now our attention to the existing experimental constraints to our free parameter $\Lambda^2 t_\Lambda$. To this end, we refer to a recent analysis [20], where it was shown that

$$\delta n_x = 0.18_{-0.41}^{+0.44} \quad (15)$$

at 95 % C.L. With the help of Eqs. (14) and (15), we immediately see that, within two standard deviations (or by taking the central value), we get the upper bound:

$$\frac{\Lambda^2 t_\Lambda}{M_P} \leq 0.42 \text{ (0.23)}. \quad (16)$$

Eq. (16) gives an experimental bound for the product of the two parameters Λ^2 and t_Λ , which are the essential ingredients of Eq. (9). If the theory is quantized at $t_\Lambda = t_P = M_P^{-1}$, one gets $\Lambda \leq 0.65 M_P$ ($0.48 M_P$). Remarkably, these results agree with the natural choice of taking Λ at t_P of the same order of magnitude of M_P .

In this respect, it is worth to notice that the choice $\Lambda = M_P$ at $t_\Lambda = t_P$, which we used in [2], falls beyond the two standard deviation limit in Eq. (16). But it must also be remarked that this too stringent limit stems from the simplest possible scenario considered here, namely a single weakly interacting scalar field. For more involved models, the numerical bound in Eq. (16) can be slightly modified.

Clearly, Eq. (16) leaves open the possibility of quantizing the EFT at later times, $t_\Lambda > t_P$, provided that the UV cut-off Λ is appropriately rescaled in order to fulfill the bound. We observe that, according to Eq. (16), Λ has to scale with t_Λ exactly in the same way as the temperature of a radiation dominated universe does with cosmic time. This indicates that, at times of the order of t_P , the temperature of the universe is directly related to the energy scale cut-off of the quantum field theories describing the universe itself, as should be expected on physical grounds.

Interestingly, the extra amount of radiation predicted in our model could be appreciable from forthcoming experiments on Cosmic Background Radiation (CBR) anisotropy, like those of the Planck satellite. Their data could be used to explore the possibility of a direct measurement of the primordial zero-point energy density of the EFT which describes physics at the Planck scale.

Our arguments, although only suggestive, clearly indicate that there is space for a non standard scenario which could explain the absence of the enormous amount of energy density predicted by a straightforward but simplistic field quantization. According to our scenario, the zero-point energy contribution to the energy density of the universe is almost entirely washed out by the cosmological evolution. After the cosmic dilution has taken place, however, there is an extremely tiny left over which could be experimentally detected in the near future as an additional contribution to the radiation energy density content of our universe.

Actually, *via* their sensitivity to the universe radiation content at the time of last scattering, new CBR data should provide constraints against which our model predicting a washing out of zero point energies could be tested. These data might signal whether the zero-point energies should be searched in a previously unsuspected sector (radiation) of the universe energy budget.

Before ending this letter, we would like to add few more comments. First of all, we would like to stress again that our momentum cutoff is not Lorentz invariant (in the more general case it would not be diffeomorphism invariant). Lorentz symmetry violating effects and related extensions of the Standard Model have been widely studied in the past years, see e.g. [21] and, in particular, it has been shown that these effects can be described within the framework of effective field theories [22]. In the effective field theory under consideration, the Lorentz violation has measurable consequences only as far as the vacuum energy density is concerned and therefore it can be entirely associated to a property of the vacuum state. From the phenomenological point of view, one should expect

to observe these Lorentz symmetry violating effects in the experimental determination of the radiation energy density, as an excess with respect to standard expectations. However, as we have shown above, these effects are very tiny and their possible detection would be the fingerprint of the preferred cosmology frame.

Moreover, it is worth to notice that, while our analysis concerns the fate of the zero point energies and points towards the cancellation of the latter due to Eq.(7), other authors have considered another important facet of the cosmological constant problem, namely the possibility of a running with the energy scale of the cosmological constant term Λ_{CC} which appears in the Einstein-Hilbert quantum gravity action [23, 24, 25].

The latter approach shows some common features with the mechanism described here. In fact, in both cases it is considered (in the framework of the Wilsonian effective theories) the running of some parameters entering the Einstein equations, Eq. (8), with an energy scale that can be eventually re-expressed in terms of the cosmic time. However, unlike these works which are focused on the running of the cosmological and Newton constants, we are only concerned with a single contribution to the energy-momentum tensor, namely the zero point energies contribution of an effective field theory. This has a different EOS (and therefore a different time evolution) with respect to the cosmological constant, with the consequence that the former decreases with time and is very tiny (and can be detected only under favourable conditions as explained above), whereas the latter, as confirmed by observations, has a much larger measured value and a different EOS, namely $w \simeq -1$.

Nevertheless, the running considered by these authors, could provide (and we believe it does) the dynamical mechanism (suppression by quantum fluctuations) which eventually should give the measured value of the CC. In this respect, we think that our analysis together with the work these authors should naturally merge in a coherent picture which could provide a scenario for the solution of the CC problem in its different aspects.

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